Highly Efficient Implementation of 'Solid-Shell' Finite Elements with Enhanced Assumed Strains in Explicit Time Integration

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Explicit time integration is commonly used in finite element analysis and perfectly suited to highly dynamic applications, e.g. crash or impact. The absence of equation solving but only vector operations on global level, leads to small CPU-time requirements per time step. However, the time step size is limited to a critical value, consequently transient analyses require a high number of time steps. This is where efficiency in the handling of the operations for force calculations in each step play the dominant role. The efficiency of the time integration scheme on global level is based on the application of diagonalized mass matrices; as a consequence the computation of the accelerations at the current time step

$$\mathbf{a}^n = \mathbf{M}^{-1} \left(\mathbf{f}^{ext,n} - \mathbf{f}^{int,n} \right) \tag{1}$$

involves only vector operations. At every time step, the internal forces $\mathbf{f}^{int,n}$ have to be integrated over all elements, thus a dominant part of the required CPU-time is spent on element level. This motivates an efficient implementation of the element routines especially for so-called explicit FE codes, which is achieved in the current project, using the specific programming tool AceGen [1], a plug-in for the computer algebra software Mathematica. The advantage of such tools is – after a first glance – the straight forward and extremely fast generation of element program variations due to the use of e.g. symbolic differentiation of equations and the error free code development at the same time. As the generated code is automatically optimized, a very efficient implementation can be achieved. Comparison of manually programmed and automatically optimized element code within the in-house FE program lead to a reduction of the necessary CPU-time of up to 90 % for several numerical examples.

The Enhanced Assumed Strain (EAS) technique, based on the proposal of Simo and Rifai [2] is applied in different forms in order to reduce artificial stiffness effects – the so-called locking phenomena. Additional degrees of freedom are introduced in order to enhance the compatible strain field with

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_k + \tilde{\boldsymbol{\varepsilon}} \quad \text{with} \quad \tilde{\boldsymbol{\varepsilon}} = \tilde{\mathbf{M}}\,\boldsymbol{\alpha}. \tag{2}$$

The matrix $\tilde{\mathbf{M}}$ represents the enhancements of the strain field and the vector $\boldsymbol{\alpha}$ contains additional degrees of freedom, which can be condensed out on element level. This requires the inversion of a square matrix with the dimension $(\dim \boldsymbol{\alpha})^2$ for all elements at every time step.

In the current contribution, a volumetric shell element with only displacement degrees of freedom as presented e.g. in [3] is implemented for explicit time integration. The so-called geometric locking effects, e.g. transverse shear locking and trapezoidal locking, are cured with the method of Assumed Natural Strains (ANS) [4, 5], which is not discussed in detail. The EAS method in different variations is applied in order to cure volumetric locking especially for the normal thickness strains E_{33}. The linearly interpolated membrane strains E_{11} and E_{22} are coupled by the Poisson ratio with E_{33}. Thus, e.g. a pure bending scenario, would lead to linear stresses S_{33} instead of $S_{33} = 0$ and consequently to an overly stiff structural behavior.

In this contribution, three formulations are presented, using different numbers of EAS parameters. The enhanced thickness strains are computed as

$$\tilde{E}_{33} = \frac{\det \mathbf{J}_0}{\det \mathbf{J}} t_{33} \, \mathbf{M}^i \, \boldsymbol{\alpha} \,, \tag{3}$$

where $\mathbf{J}_0$ is the determinant of the Jacobian $\det \mathbf{J}$, evaluated at the element mid point and t_{33} transforms $\mathbf{M}^i$ to the local element co-ordinate system. Three formulations were implemented with the matrices

$$\mathbf{M}^1 = [\ \zeta\ ]\,, \qquad \mathbf{M}^3 = [\ \zeta \quad \xi\zeta \quad \eta\zeta\ ] \qquad \text{and} \qquad \mathbf{M}^4 = [\ \zeta \quad \xi\zeta \quad \eta\zeta \quad \xi\eta\zeta\ ] \tag{4}$$

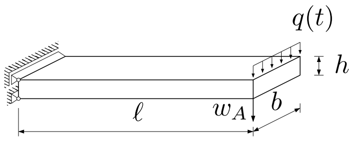
respectively. For the presented elements with linear in-plane and thickness interpolation of geometry and displacements, a “full” Gauß integration rule with 2 x 2 x 2 integration points was chosen, in order to numerically integrate the internal forces.

The three element formulations are compared here on a benchmark for volumetric locking, well-known from static analyses – the clamped cantilever beam with tip load. As the boundaries are chosen statically determinate and geometrical locking effects are cured, the tip displacement w_A must be invariant against the Poisson ratio ν. As shown in figure 1 and known from statics, three EAS-parameters are sufficient for correct results in this example; a discussion on the performance of the different formulations for more general cases will follow. For explicit time integration further efficiency improvements can be achieved with a reduced integration for the ‘Solid-Shell’ element together with stabilization against non-physical kinematics as e.g. proposed for implict applications in [6, 7] and applied for commercial programs for the standard elements. This is in the focus as a next implementation step.

References

[1] `http://www.fgg.uni-lj.si/Symech/`, J. Korelc, 2009.

[2] *A class of mixed assumed strain methods and the method of incompatible modes*, J.C. Simo, M.S. Rifai, Int. J. Numer. Methods Eng., 29(8): 1595-1638, 1990.



$$
\begin{aligned}
E &= 2.10 \cdot 10^5 & \ell &= 10\\
\nu &= 0.0/0.3/0.499 & b &= 1\\
\rho &= 7.85 \cdot 10^{-9} & h &= 0.1\\
q(t) &= \begin{cases} 100\, q_0 & 0.00 \le t \le 0.01 \\ q_0 & 0.01 < t \le 0.02 \end{cases} & q_0 &= 1
\end{aligned}
$$

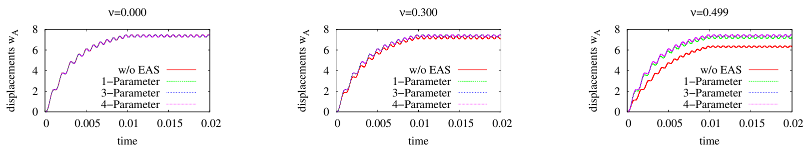


Figure 1: Test for volumetric locking; Cantilever with tip load, varying the POISSON ratio

[3] *A systematic development of 'solid-shell' element formulations for linear and non-linear analyses employing only displacement degrees of freedom*, R. Hauptmann, K. Schweizerhof, Int. J. Numer. Methods Eng., 42(1): 49-69, 1998.

[4] *A formulation of general shell elements - the use of mixed interpolation of tensorial components*, K.-J. Bathe, E.N. Dvorkin, Int. J. Numer. Methods Eng., 22(3): 697-722, 1986.

[5] *An assumed strain approach avoiding artifical thickness straining for a non-linear 4-node shell element*, P. Betsch, E. Stein, Commun. Numer. Methods Eng., 11: 899-909, 1995.

[6] *Enhanced assumed strain (EAS) and assumed natural strain (ANS) methods for one-point quadrature solid-shell elements*, R.P.R. Cardoso, J.W. Yoon, M. Mahardika, S. Choudhry, R.J. Alves de Sousa, R.A. Fontes Valente, Int. J. Numer. Methods Eng., 75(2): 156-187, 2008.

[7] *A reduced integration solid-shell finite element based on the EAS and the ANS concept – Geometrically linear problems*, M. Schwarze, S. Reese, Int. J. Numer. Methods Eng., online, 2009.